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7.1: Introduction to Decimals

Definition: A decimal number is a notation to represent the sum of a whole number and fractions whose denominator is a power of 10.

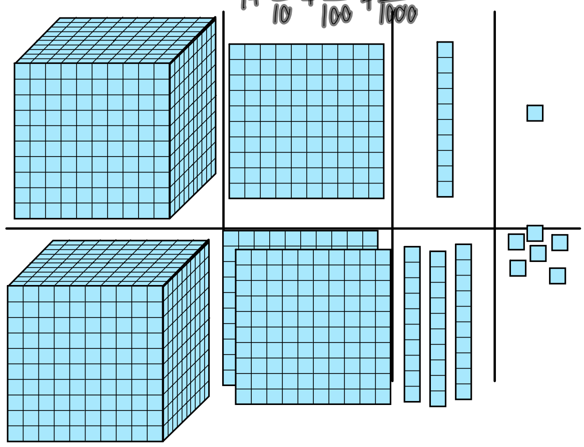
Example: Represent $1 + \frac{3}{10} + \frac{6}{100} + \frac{2}{1000}$ as a decimal number.

1.362

Definition: The " . " above is called the decimal point.

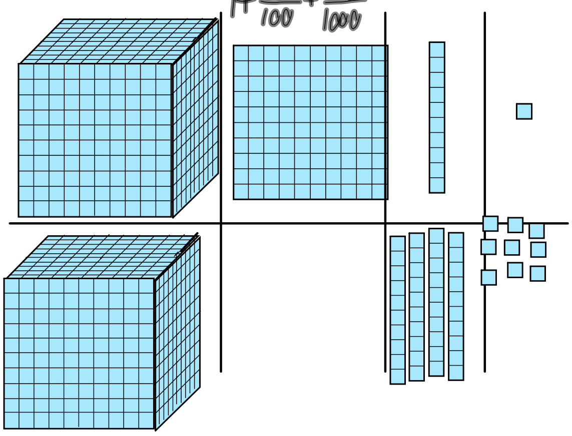
Example: Write the number 1.236 as a sum of fractions, then represent it using base 10 blocks.

$1 + \frac{2}{10} + \frac{3}{100} + \frac{6}{1000}$



Example: Write the number 1.049 as a sum of fractions, then represent it using base 10 blocks.

$1 + \frac{4}{100} + \frac{9}{1000}$



Every place value is named after the denominator of its corresponding fraction.

Example: Circle the hundredths and ten-thousandths place in the following number.

3.14159

Question: Why does it not matter if we write additional zeroes at the end of a decimal number? (For example, 1.500 = 1.5)

0 is a placeholder but it's not holding a place

$1 + \frac{5}{10} + \cancel{\frac{0}{100}} + \cancel{\frac{0}{1000}} = 1.5$

A more standard interpretation is to represent the fraction as the whole number plus the entire decimal over a common denominator.

Example: Write 16.23 in this manner.

$16 + \frac{2}{10} + \frac{3}{100}$

$16 + \frac{20}{100} + \frac{3}{100} = 16 + \frac{23}{100}$

16.33

$\frac{30}{100}$

$16 + \frac{3}{10} + \frac{3}{100}$

$16 + \frac{33}{100}$

Example: Write 1.0495 in this manner.

$1 + \frac{495}{10000}$ ← consider the name of the last place value

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$$13.073 = 13 + \frac{73}{1000}$$

$$13 + \frac{7}{100} + \frac{3}{1000}$$

$$\frac{70}{1000}$$

A decimal number is read by saying the whole number, "and" the decimal part as a fraction.

Example: Write a out the way to read the following numbers.

(a) 16.23 $= 16 + \frac{23}{100}$

Sixteen and twenty-three hundredths

(b) 1.0495

One and four hundred ninety-five ten-thousandths

Example: Write each of the following as a fraction in simplest form.

(a) 0.625

$$= \frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \boxed{\frac{5}{8}}$$

(b) 1.42

$$= 1 + \frac{42}{100} = \boxed{1\frac{21}{50}} \text{ or } \frac{71}{50}$$

(c) 0.1144 $\frac{1144}{10000} = \frac{286}{2500} = \boxed{\frac{143}{1250}}$

Example: Write each of the following as a decimal.

(a) $\frac{625}{10000} = 0.0625$

(b) $\frac{11}{125} = \frac{11}{5^3} \cdot \frac{2^3}{2^3} = \frac{88}{10^3} = 0.088$

(c) $\frac{27}{40} = \frac{27}{2^3 \cdot 5} \cdot \frac{5^2}{5^2} = \frac{675}{10^3} = 0.675$

(d) $\frac{1}{32} = \frac{1}{2^5} \cdot \frac{5^5}{5^5} = \frac{3125}{10^5} = 0.03125$

Definition: A terminating decimal is a decimal that can be written with a finite number of digits after the decimal point.

Example: Try to write $\frac{1}{3}$ as a decimal number.

$$\frac{1}{3} \cdot \frac{?}{?} = \frac{\sim}{10^k}$$

$$3 \cdot ? = \underbrace{10^k}$$

has no factor of 3, so cannot be done

Theorem: A rational number $\frac{a}{b}$ in simplest form can be written as a terminating decimal if and only if the prime factorization of the denominator contains no primes other than 2 or 5.

~~Proof:~~

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Example: Which of the following can be written as terminating decimals?

(a) $\frac{1}{8} = \frac{1}{2^3} \cdot \frac{5^3}{5^3} = \frac{125}{10^3} = .125$

(b) $\frac{8}{675} = \frac{8}{5^2 \cdot 3^3}$

(c) $\frac{25}{98} = \frac{25}{2 \cdot 7^2}$

no, not terminating

(d) $\frac{22}{275} = \frac{\cancel{22}^{\,2}}{5^2 \cdot \cancel{11}_{\,1}} = \frac{2}{5^2} \cdot \frac{2^2}{2^2} = \frac{8}{10^2} = .08$

$\frac{3}{20} = \frac{3}{2^2 \cdot 5} \cdot \frac{5}{5} = \frac{15}{2^2 \cdot 5^2} = \frac{15}{10^2} = .15$

$2 \cdot 2 \cdot 5 \mid \cdot 5$ $\quad 2 \cdot 2 \cdot 5 \cdot 5$ $\quad 10 \cdot 10$

$\frac{7}{50} = \frac{7}{2 \cdot 5^2} \cdot \frac{2}{2} = \frac{14}{10^2} = .14$

$2 \cdot 5 \cdot 5$